



Rewarding Learning

General Certificate of Secondary Education

Further Mathematics

Unit 4

Discrete and Decision Mathematics

[GFM41]

Assessment

MARK SCHEME

GCSE MATHEMATICS

General Marking Instructions

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for accurate working, whether in calculation, reading from tables, graphs or answers.

MW indicates marks for combined method and accurate working.

Assessment Objectives

Below are the assessment objectives for GCSE Further Mathematics.

Use and apply standard techniques (AO1)

Candidates should be able to:

- accurately recall facts, terminology and definitions;
- use and interpret notation correctly; and
- accurately carry out routine procedures or set tasks requiring multi-step solutions.

Reason, interpret and communicate mathematically (AO2)

Candidates should be able to:

- make deductions, inferences and draw conclusions from mathematical information;
- construct chains of reasoning to achieve a given result;
- present arguments and proofs; and
- assess the validity of an argument and critically evaluate a given way of presenting information.

Solve problems within mathematics and in other contexts (AO3)

Candidates should be able to:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
- make and use connections between different parts of mathematics;
- interpret results in the context of the given problem;
- evaluate methods used and results obtained; and
- evaluate solutions to identify how they may have been affected by assumptions made.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be **followed through** from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

It should be noted that where an error trivialises a question, or changes the nature of the skills being tested, then as a general rule, it would be the case that not more than half the marks for that question or part of that question would be awarded; in some cases the error may be such that no marks would be awarded.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

- (a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;
- (b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier, only a proportion of the marks will be available (based on the professional judgement of the examiner).

Additional guidance for teachers

These notes explain how the marks allocated in the published mark scheme are to be applied.

In the mark scheme, M indicates method marks and W indicates work marks. MW indicates a combined method and work mark. Work marks should not be awarded if the method is incorrect.

If a candidate misreads a question, eg copies a given equation incorrectly, deduct 1 mark and then FT (follow through), as long as the question is not made easier. However, work marks should not be awarded to answers which are inconsistent with the question, eg negative numbers of people.

If a correct answer in working is transferred incorrectly (or not at all) to the answer line, give BOD (benefit of doubt). However, if the incorrect answer is used in a subsequent section, penalise one mark in the subsequent section and treat the remaining work as a misread.

As a general rule, ignore work that is scored out.

However, if correct working is scored out, but subsequently used correctly, give BOD.

If incorrect working is not scored out but subsequently corrected, give BOD.

If more than one attempt at a question is made and none is scored out, mark the attempt that corresponds to the answer given in the answer line. If no answer is given in answer line, mark the worst attempt, unless the better attempt is clearly the one that is intended to be taken.

If incorrect or unnecessary working is given after a correct answer, ignore this if the correct answer is on the answer line. However, if a subsequent incorrect answer is given on the answer line deduct 1 mark.

Mark answers only in appropriate section. Allow forward marking if carried through into appropriate section, eg if working for part (ii) is given in part (i) and not repeated but used in part (ii), give appropriate marks in part (ii).

Do not allow backward marking, eg answer to part (ii) is given as answer to part (i) and no answer, or an incorrect answer, is given in part (ii).

All working must be shown, so answers with no working can get no marks, unless the working is trivial.

Answers should be exact where possible.

Unless specifically stated in the mark scheme, accept one or more dp (decimal places), or 3 significant figures, unless insufficient accuracy leads to very inaccurate or meaningless answer.

Ignore slight rounding errors, eg arising from calculations using values to 2 rather than 3 or more places. Accept 1.5 instead of 1.50 for an answer required to 2 dp. Ignore truncation error in the last digit unless a specific accuracy is required, eg 0.345 truncated to 0.34.

1.
 - (i) W1 only one work mark for the correct answer
 - (ii) W1 W1 only one work mark for each correct answer
 - (iii) W1 only one work mark for the correct answer
 - (iv) MW1 for a line through (0, 80) and (80, 0) shaded above the line
 MW1 for both lines $x=20$ shaded left of the line and $y=15$ shaded below the line
 MW1 for a line through (0, 30) and (48, 0) shaded below the line
 Note allow all 3 marks if the correct area is alternatively indicated viz:
 - (1) if all 4 lines are shaded on the opposite sides, or
 - (2) if there is no shading but the correct area is marked with a capital R
 - (v) MW1 for the correct expression $P = 60x + 50y$ or $P = 50y + 60x$
 - (vi) M1 for trying to consider the value of P at both (65, 15) and (20, 60)
 W1 for both 4650 and 4200, the correct values of P at the 2 points above,
 or for correct values of their (incorrect) P expression at both above points
 M1 for maximum value of P in the solution region is £4650 (no other answer allowed)
 (This may simply be indicated by a tick against the correct value.)

2.
 - (a) M1 for attempting to calculate 20 choose 15.
 W1 for 15504
 - (b) Method 1
 M1 for use of the multiplicative principle – attempting $11 \times 10 \times 9$
 W1 for 990
 MW1 for correctly evaluating 8 choose 3 = 56
 MW1 for correctly multiplying together their 2 answers to the two parts above
 → Method 2
 → MW1 for correctly evaluating 11 choose 3 = 165
 → M1 for use of the multiplicative principle – attempting $8 \times 7 \times 6$
 → W1 for 336
 → MW1 for correctly multiplying together their 2 answers to the two parts above

- 3
 - (i) M1 for the use of FOUR point moving averages
 W1 for 6 correct moving averages / second W1 for all 9 correct.
 - (ii) M1 for correctly plotting 6 of the points / second M1 for all 9 correct
 W1 for a good attempt at a line of best fit
 (This should be drawn close to both end points and at least one interim point both above and below).
 - (iii) M1 for attempting to evaluate an average of the last 3 points with x
 W1 for setting this average to 26.75 or their reading for the number of drivers taken from their line of best fit.
 W1 for the correct solution to their equation
 Final sentence in context is not necessary for the final mark.

- 4 (i) MW1 for the correct Boolean expression only
(ii) MW1 for both of the expressions in brackets.
One must equal “p and not q” while the other can be either the correct answer or their answer to part (i).
W1 for joining both expressions with “or”
(iii) Marking of first table:
M1 for attempting to fill in the table and both “not p” and “not q” columns correctly
W1 for the “p and not q” column correct
W1 for the expression from (i) column correct
W1 for the disjunction (“or”) of the two previous columns together column correct.
Marking of second table:
W1 for two correct columns from the first three response columns correct
W1 second mark is given for all of the first three response columns correct
W1 this mark is only given for the final column if it is the conjunction (“and”) of the previous two columns, and it is identical to the final column in the first table.
5. (i) M1 W1 for attempting the product of three combinations
W1 for the correct answer (4200)
M1 for attempting the product of three combinations
W1 for the correct answer (5880)
W1 for the sum of their two previous triple products.
6. (i) Critical path diagrams with errors are difficult to evaluate.
We adopt the following structure for both forward and backward passes:
- one mark for times calculated from individual in-flows
 - one mark for all the remaining times
- Thus
MW1 for early times 6, 5, 9, 8 all correct
MW1 for remaining early times all correct: 13, 16, 16, 22
MW1 for late times 22, 16, 13, 11, 18 all correct
MW1 for remaining late times all correct: 7, 5, 11
- (ii) MW1 for correct answer or (taken from their diagram)
[late time from end node of F] – [early time from start node of F] – [duration of F]
MW1 for correct answer or (taken from their diagram)
[late time from end node of H] – [early time from start node of H] – [duration of H]
- (iii) MW1 only awarded for the correct critical path ie B, E, I, L
MW1 this mark may be given for the correct answer (22 weeks – units not needed)
or the correctly evaluated length of whatever critical path they describe,
– as long as it is a path from start to finish.

1 (i) $x + y \leq 80$

W1

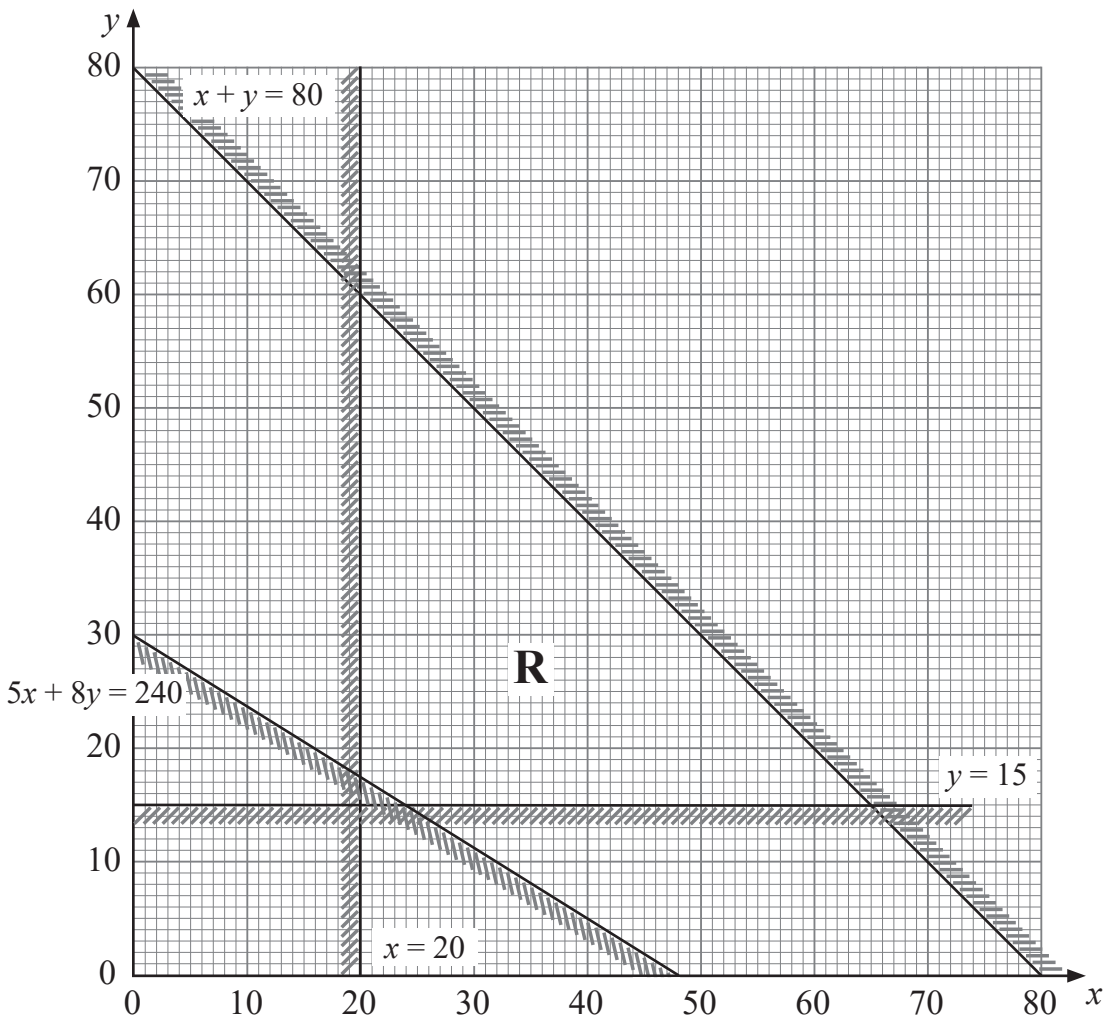
(ii) $x \geq 20, y \geq 15$

W1 W1

(iii) $5x + 8y \geq 240$

W1

(iv)



Line (0, 80) to (80, 0) and correct side shaded
 Lines $x = 20$ and $y = 15$ and correct sides shaded
 Line (0, 30) to (48, 0) and correct side shaded

MW1

MW1

MW1

AVAILABLE
MARKS

		AVAILABLE MARKS
(v)	$P = 60x + 50y$	MW1
(vi)	Key points in R are (65, 15) and (20, 60)	M1
	[(24, 15) and (20, 18) are clearly not needed]	
	(65, 15) gives $60 \times 65 + 15 \times 50 = 4650$	
	(20, 60) gives $60 \times 20 + 60 \times 50 = 4200$	W1
	maximum profit = £4650	M1
2	(a) 15 students from 20 $\rightarrow {}_{20}C_{15} = 15\,504$ ways	M1 W1
	(b) Captain, Secretary and Chairperson can be found in	
	$11 \times 10 \times 9 = 990$ ways	M1 W1
	Then 3 Prefects out of the remaining 8 Mentors can be found in	
	${}_8C_3 = 56$ ways	MW1
	Total number of ways = $990 \times 56 = 55\,440$ ways	MW1
<p>Alternative Solution</p> <p>3 Prefects from 11 Mentors can be found in</p> <p>${}_{11}C_3 = 165$ ways</p> <p>From remaining 8 mentors, Captain, Secretary and Chairperson can be found in</p> <p>$8 \times 7 \times 6 = 336$ ways</p> <p>Total number of ways = $165 \times 336 = 55\,440$ ways</p>		
		11
		6

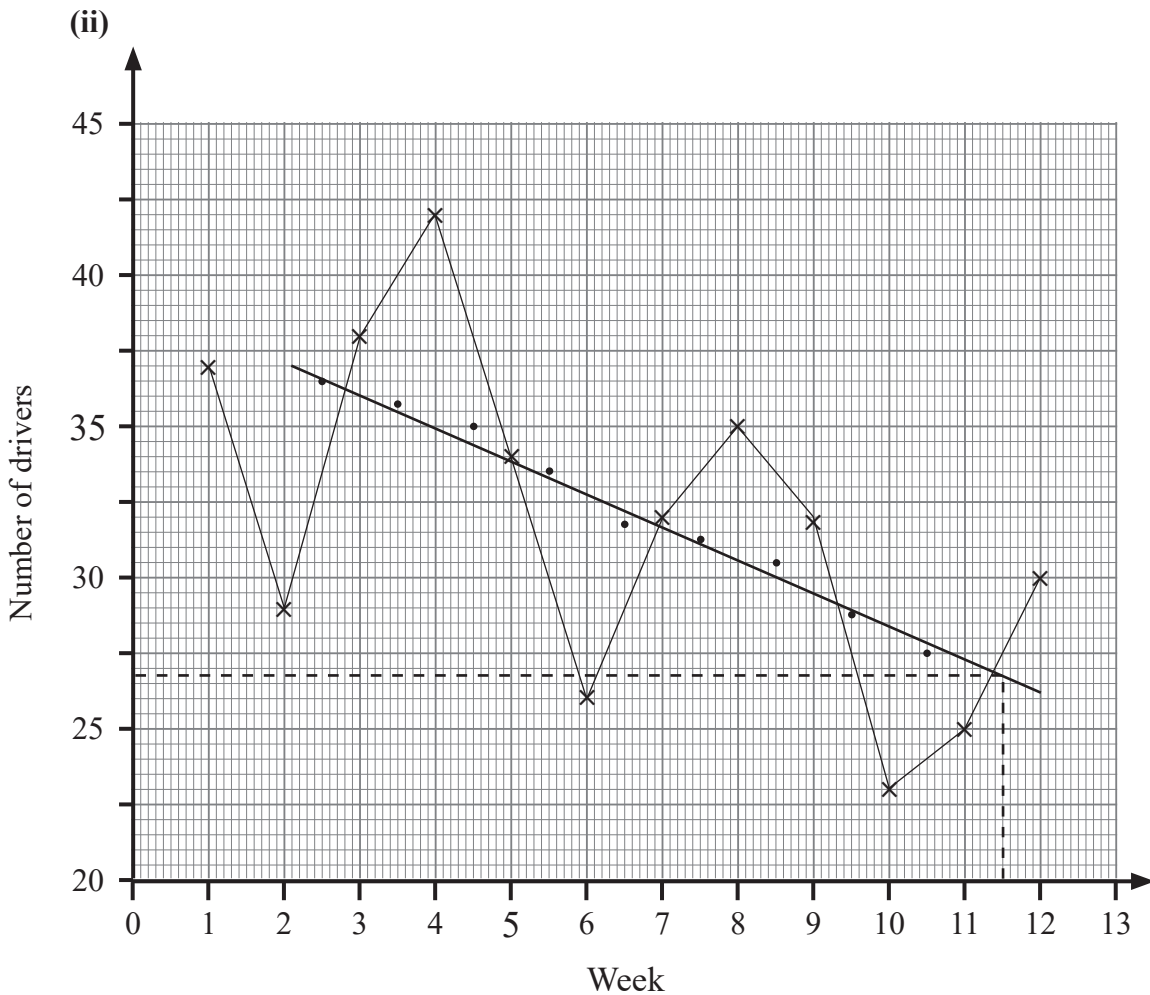
3 (i) 4-point moving averages

M1

- 37
- 29 → 36.5
- 38 → 35.75
- 42 → 35
- 34 → 33.5
- 26 → 31.75
- 32 → 31.25
- 35 → 30.5
- 32 → 28.75
- 23 → 27.5
- 25
- 30

W2

AVAILABLE MARKS



Set M2
Line W1

(iii) $\frac{30 + 25 + 23 + x}{4} = 26.75$

M1 W1

$x = 29$

W1

29 drivers likely to be stopped

4 (i) q and not p

MW1

(ii) $(p$ and not $q)$ or $(q$ and not $p)$

both expressions correct MW1
joined by “or” W1

(iii)

					Expression from (i)	Expression from (ii)
p	q	not p	not q	p and not q	q and not p	$(p$ and not $q)$ or $(q$ and not $p)$
T	T	F	F	F	F	F
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	F	F

p	q	p and q	not $(p$ and $q)$	p or q	not $(p$ and $q)$ and $(p$ or $q)$
T	T	T	F	T	F
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	F	F

Columns not p , not q M1

Columns p and not q } W1
 q and not p } W1

Final top column W1

Columns p and q } W2
not $(p$ and $q)$ }
 p or q }

Last column W1

AVAILABLE MARKS

9

10

5 Either ${}^6C_2 \times {}^7C_3 \times {}^8C_1$
 $= 15 \times 35 \times 8 = 4200$ ways

M1 W1

or ${}^6C_1 \times {}^7C_4 \times {}^8C_2$

W1

$= 6 \times 35 \times 28 = 5880$ ways

M1

W1

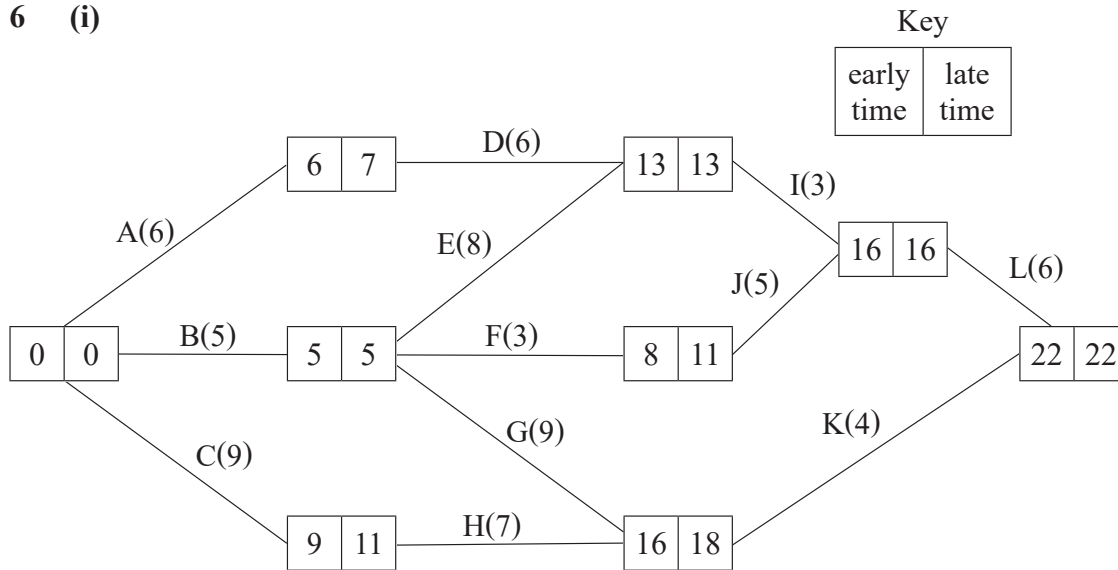
Total number of ways $= 4200 + 5880$
 $= 10080$ ways

W1

AVAILABLE
MARKS

6

6 (i)



Early times	6, 5, 9, 8	MW1
Early times	13, 16, 16, 22	MW1
Late times	22, 16, 13, 11, 18	MW1
Late times	7, 5, 11	MW1

(ii) Float time of F = $11 - 5 - 3 = 3$ weeks
 Float time of H = $18 - 9 - 7 = 2$ weeks

MW1

MW1

(iii) Critical path: B, E, I, L

MW1

Path length: 22 weeks

MW1

8